Theory Q3 MARKING SCHEME: TIPPE TOP

Asian Physics Olympiad Adelaide 2019

Marks in brackets are partial marks, and do not count toward totals.

Negative marks in brackets are relative to full marks available for that portion, i.e. relative to unbracketed line above. (full) is equivalent to (-0.0).

ecf = 'error carried forward'

Question	Required answer	Total Marks	Given Marks
A.1 **(1.0)**			
	$\mathbf{F}_{\text{ext}} = (N - mg)\hat{z} + \mathbf{F}_f$ or equivalent	0.2	
	($\mathbf{F}_f$ expanded but restricted to $\hat{\mathbf{x}}$ or $\hat{\mathbf{y}}$ direction)	(-0.1)	
	free body diagrams with $\mathbf{F}_g$, $N\hat{\mathbf{z}}$, $\mathbf{F}_f$ in right directions (direction of $\mathbf{F}_f$ in xz-plane diagram does not matter	0.6	
	($\mathbf{F}_f$ drawn in $\hat{\mathbf{x}}$ or $\hat{\mathbf{y}}$ direction but $\mathbf{F}_f$ term correct in expression above)	(full)	
	($\mathbf{F}_f$ drawn in $\hat{\mathbf{x}}$ or $\hat{\mathbf{y}}$ direction and $\mathbf{F}_f$ term expanded but restricted to $\hat{\mathbf{x}}$ or $\hat{\mathbf{y}}$ direction)	(-0.1)	
	(only one correct diagram)	(-0.2)	
	$\mathbf{v_A}$ in $-\mathbf{F}_f$ direction (must be within XY plane)	0.2	
A.2 **(0.8)**			
	$\boldsymbol{\tau}_{\text{ext}} = \mathbf{a} \times (N\hat{\mathbf{z}} + \mathbf{F}_f)$	0.3	
	(total force at point A wrong but consistent with A.1)	(ecf 0.3)	
	(have $\boldsymbol{\tau}_{\text{ext}} = \mathbf{a} \times \mathbf{F}_A$ but force at A wrong)	(0.2)	
	correct $\mathbf{a} = \alpha R\hat{\mathbf{3}} - R\hat{\mathbf{z}}$	0.2	
	getting to final answer $\sum \boldsymbol{\tau}_{\text{ext}} = RF_{f,y}(1 - \alpha\cos\theta)\hat{\mathbf{x}} + [RF_{f,x}(\alpha\cos\theta - 1) - \alpha RN\sin\theta]\,\hat{\mathbf{y}} + \alpha R\sin\theta F_{f,y}\hat{\mathbf{z}}$	0.3	
	(partial credit if started off right but cross/dot products wrong)	(0.1-0.2)	
A.3 **(0.4)**			
	recognising $\mathbf{v_A} \cdot \hat{\mathbf{z}}$ is needed	0.1	
	$\mathbf{v_A} = \dot{\mathbf{s}} + \boldsymbol{\omega} \times \mathbf{a}$ stated	0.2	
	differentiating contact condition in xyz or XYZ frame	0.1	
A.4 **(0.8)**			
	$\boldsymbol{\omega} = \dot{\theta}\hat{\mathbf{2}} + \dot{\phi}\hat{\mathbf{z}} + \dot{\psi}\hat{\mathbf{3}}$	0.4	
	(implicitly recognising total angular velocity is sum of several angular terms)	(0.1)	
	(each correct angular term)	(0.1)	
	(above if not written explicitly, but answer is correct in either xyz or 123 frame)	(full)	
	$\hat{2}, \hat{3}$ to xyz frame correct	0.2	
	$\hat{\mathbf{z}}$ to 123 frame correct	0.2	
	Note: credit for these transformations can be given in other parts (e.g. **A.5**) if not allocated here		
A.5 **(1.0)**			
	$E_T = K_T + K_R + U_G$	0.3	
	$U_G = mgR(1 - \alpha\cos\theta)$	0.1	
	$K_R = \frac{1}{2}(I_1(\dot{\phi}^2\sin^2\theta + \dot{\theta}^2) + I_3(\dot{\psi}) + \dot{\phi}\cos\theta)^2$	0.3	
	(dimensionally correct term in form of $\sum I\boldsymbol{\omega}^2$)	(0.1)	
	$K_T = \frac{m}{2}\left[(v_y - R\sin\theta(\alpha\dot{\phi} + \dot{\psi}))^2 + \dot{\theta}^2\alpha^2R^2\sin^2\theta\right]$	0.3	
A.6 **(0.4)**			
	taking dot product for z-component	0.1	

	correct result from **A.2** **Note:** no credit given for $\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau}_{\mathbf{ext}}$ here, but can allocate those marks for **A.10** for work here	0.3	
A.7 **(1.4)**			
	U_G increases as centre of mass rises	0.2	
	(above implied in figure in **A.8**)	(full)	
	$K_T \sim 0$ at start and finish	0.2	
	(above implied in figure in **A.8**)	(full)	
	therefore energy transfer from K_R to U_G	0.2	
	normal force does no work	0.1	
	identify y-component of $\mathbf{F}_f$	0.4	
	(only stating friction force)	(0.2)	
	correct value of $\frac{E_T}{dt}$	**0.3**	
	Note: This was 0.2 in previous version of marking scheme, which gave the wrong total		
A.8 **(2.0)**			
	E_T monotonically decreasing	0.2	
	E_T constant from **IV** to **V**	0.2	
	U_G rising from **I** to **IV**	0.2	
	U_G constant from **IV** to **V**	0.2	
	$K_T \sim 0$ at **I**	0.2	
	$K_T = 0$ at **V**	0.2	
	K_T increases then decreases between **I** and **V**	0.2	
	K_R monotonically decreasing	0.2	
	$K_R = 0$ at **V**	0.2	
	K_R decreasing while U_G is rising	0.2	
	Note: scale of U_G including at start does not matter. Labels in solutions not required. (a)–(e) in solutions changed to **I**–**V** on answer sheet.		
A.9 **(0.5)**			
	$\mathbf{L} = \mathbf{I}\boldsymbol{\omega}$ where $\mathbf{I}$ is clearly the inertia tensor	0.1	
	correct $\mathbf{L} \times \hat{3}$ in 123 frame	0.2	
	(above done with $\mathbf{L}$ and $\boldsymbol{\omega}$ in 123 frame, without full expansion of L_1, L_2 terms)	(full)	
	$k = I_1$ (can be implied)	0.2	
A.10 **(1.7)**			
	$\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau}_{\text{ext}}$	0.3	
	$\therefore \frac{d\mathbf{L}}{dt} \cdot \mathbf{v} = \boldsymbol{\tau}_{\text{ext}} \cdot \mathbf{v}$ for any vector $\mathbf{v}$	0.2	
	$\boldsymbol{\tau}_{\text{ext}} \perp \mathbf{a}$ or other valid argument	0.3	
	$\therefore \frac{d\mathbf{L}}{dt} \cdot \mathbf{a} = 0$	0.2	
	$\mathbf{v} = \mathbf{a}$ or any scalar multiple of $\mathbf{a}$	0.3	
	demonstrating $\dot{\lambda} = 0$	0.4	