

Theory

Q1

RF reflectometry for spin readout for silicon quantum computing

Marking scheme. Version 1.1

Question part	Total marks	Partial marks	Explanation for partial marks and special cases
A.1	0.2	0.1 0.1	Velocity dimensionally correct and is less than $c$ Correct final answer <ul style="list-style-type: none"> <li>0.2 total for correct answer with no justification</li> </ul>
A.2	0.2	0.1 0.1	Applying Gauss theorem Correct final answer
	0.3	0.1 0.1 0.1	Capacitance formula Electric potential formula Correct final answer
A.4	0.3	0.1 0.1 0.1	Bio-Savart law or equivalent for magnetic field of a wire Inductance formula Correct final answer
A.5-i	0.8	0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1	Adding one extra $\delta L, \delta C$ link does not change the semi-infinite wire Equating impedance the circuit with one extra link to $Z_0$ Sum of impedance in parallel ( $L$ and $Z_0$ ) Sum of impedance in series Correct equation for $Z_0$ Relating $\delta L, \delta C$ to $L_x, C_x$ $\delta L \rightarrow 0$ limit simplification Correct final answer
A.5-ii	0.2	0.1 0.1	Correct b/a formula Correct final answer
B.1	1	0.1 0.2 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1	$Z_0$ in terms of $L_x, C_x$ Method of images Magnetic flux and its relation to $L_x$ Adding B-fields of the real and the imaginary wires Correct $L_x$ Potential and its relation to $C_x$ Adding E fields of two wires Capacitance per length $C_x$ Final result for $Z_0$
C.1	1	0.1 0.1 0.1 0.1 0.2 0.1 0.1 0.1 0.1	Starting with a workable approach (would lead to answer if followed through) Concept of equating voltage amplitudes Correct voltage matching equation Current conservation concept Current conservation equation (-0.1 if signs are wrong) Apply Ohm's law Correct equation to solve for $V_r/V_i$ Solving the equation Final answer in term of $\Gamma$ <ul style="list-style-type: none"> <li>Proof of work required. Max 0.2 for stating the answer with no proof</li> </ul>
C.2	0.2	0.1 0.1	$\pi$ -shift of reflected signal implies $\Gamma < 0$ Correct condition stated

D.1(i)	1	0.1 0.3 0.3 0.3	A variety of approaches acceptable if lead to correct answer Charge on QD equals $-ne$ Treat charge on the QD as the charge on $C_g$ (neglecting tunnel junctions) Voltage drop across capacitor computed correctly Equation for $\varphi_n$ in terms of sum of voltages (-0.1 if signs are wrong) <ul style="list-style-type: none"> <li>-0.1 if the term with <math>\frac{1}{2}</math> is missing</li> </ul>
D.1(ii)	0.5	0.1 0.3 0.1	A variety of approaches acceptable if lead to correct answer Relation between energy and potential Correct intermediate formula for $\Delta E_n$ Correct final answer
D.2	0.5	0.2 0.1 0.2	A difference of $\Delta E_n$ and $\Delta E_{n+1}$ considered Use the formula from D.1(ii) for $\Delta E_n$ . Correct final answer
D.3	0.5	0.2 0.2 0.1	$k_B T$ identified the relevant thermal energy $E_c$ identified as the relevant scale for electron energy Correct final answer <ul style="list-style-type: none"> <li>no penalty for numerical prefactors of order 1 or using <math>\ll</math> instead of <math>&lt;</math></li> </ul>
D.4	0.8	0.2 0.3 0.1 0.2	$\tau \sim R_t C_t$ tau on order of RC $h/\tau$ is identified as relevant scale for the fluctuation energy $E_c$ identified as the relevant scale for electron energy Correct comparison sign and correct final answer <ul style="list-style-type: none"> <li>answer for <math>\tau</math> without justification acceptable</li> <li>no penalty for numerical prefactors of order 1 or using <math>\ll</math> instead of <math>&lt;</math></li> </ul>
E.1	0.2	0.1 0.1	At least one of $\Gamma$ computed correctly Correct final answer
E.2	0.8	0.1 0.1 0.2 0.1 0.1 0.1 0.1	Understand $\Delta\Gamma \sim 1$ requires change in $Z_{tot}$ on the order of $Z_0$ Identifying OFF state as an LC circuit Choosing $L_0$ from the LC resonance condition Calculate $\Gamma_{OFF}$ Correct $Z_{tot}$ for ON state Correct numerical answer for $L_0$ Correct numerical answer for $\Gamma_{ON}$
F.1	1	0.1 0.2 0.2 0.3 0.2	Calculate $Z_{tot}$ for OFF state Choosing $\omega_{rf}$ to match a resonance Calculate $Z_{tot}$ for ON state Connect $\Delta\Gamma \sim 1$ with $Z_{tot}$ values Correct calculation and final answer
F.2	0.5	0.1 0.2 0.1 0.1	Diagram with the added element is functional Capacitance in series with the rest of the circuit Demonstrate that the added element leads to $\Delta\Gamma \sim 1$ Correct formula characterizing the added element